

# Voltage Stability Analysis

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**Abstract**— Voltage Stability is a serious concern of today power system for its secure and reliable operation. Power system stability is dependent mainly on the degree of maintaining the synchronism of the whole system. Out of the all blackouts round the globe the primary reasons for the blackouts are observed to be voltage collapse. There is a need to perform studies to ensure that the reliability of the power system is maintained at all system condition and its different operating horizons. This paper analyzes the voltage stability of a system by finding a continuum power flow solutions with base load and obtaining steady state voltage stability limit the critical point. The in-between result of the process is used to determine voltage stability index and further to identify portion of the system prone to such voltage collapse

**Index Terms**—Critical Line, Critical Power Flow Path, Line voltage Stability Index, Path Voltage Stability Index,

## 1 INTRODUCTION

WHILE planning and operating today's power systems, the voltage stability is of major and growing concern. The Power transmission utility requirements have changed considerably after the deregulation of the power industry. These changes had brought a considerable unacceptable poor quality power, which is apparent by continuous increase in sophisticated generation, transmission, distribution and service industries. The modern society where the prime energy source is electricity, the user does not tolerate power outages and other disturbances that impact their conveniences or life style. Social, environmental, right of way costs are aggregated by potential problems that hinder the construction of new transmission lines. Introduction of the deregulated energy market has further stressed the transmission grid because of maximized financial returns with minimum investment and deliver energy at a reasonable cost to the ultimate customer. One of the major problems associated with the today stressed system is voltage instability or voltage collapse. Voltage collapse is a process, which leads to a reduction of voltage in a significant portion of a power system. The tripping of transmission or generating equipment often triggers voltage collapse. [1]

Voltage stability is the ability of a power system to maintain voltage irrespective of the increase in load admittance and load power resulting in control of power and voltage. The process by which voltage instability leads to the loss of voltage in a significant part of a power system is called voltage collapse. The ability of a power system to operate not only in stable conditions, but also to remain stable following any reasonable contingency or adverse system change is termed a voltage security.

A system enters into the unstable state when a disturbance (load increase, line outage or other system changes) causes the voltage drop quickly or to drift downward and, and automatic system controls fail to improve the voltage level. The voltage decay can take a few seconds to several minutes.

Voltage stability or voltage collapse has become a major concern in modern power systems. In deregulated market conditions, a power system is set to operate at its maximum operating limits to better utilize existing facilities. This kind of system cannot withstand for any network outage. So, it is important to study the system behavior in the case of prolonged overload or any other system disturbances. [1]

## 2 TYPES OF VOLTAGE STABILITY

### 2.1 Classification of Voltage Stability

Voltage stability or voltage collapse deals with the ability of a power system to maintain acceptable voltage levels at all buses in the system in any condition whether it is normal or during disturbance. A heavily loaded system enters a state of voltage instability due to a sudden large disturbance or a change in system condition. It causes a progressive and uncontrollable decline in voltage. The main factor causing voltage instability in any power system is the inability of the system to meet its sudden growing demand for reactive power.

The two different approaches available as a tool to analyse the voltage collapse problem in a system are:

- (a) The static approach and
- (b) The dynamic approach.

(a) *Static methods involve the static model of power system components. These methods are important when the power system is in operation and planning stages, in-order to prepare an adequate fool proof plan for meeting the power requirements during different types of contingencies arising during its operation.*

(b) *The dynamic methods use time domain simulations to reveal the voltage collapse mechanism such as why and how the voltage collapse occurs. Dynamic methods analyze the effect of dynamic loads, on load tap changes (OLTC), generator over excitation limiters (OXL) on voltage collapse. [1]*

In most of the cases, the system dynamics affecting voltage stability are quite slow. The static approach effectively analyzes most of the problems. It can examine the viability of a specific operating point of the power system. In addition, static analysis method provides information such as sensitivity or degree of stability and involves the computation of only algebraic equations. It is much more efficient and faster than dynamic approaches. The static analysis approach is more attractive than the dynamic method and well suited to voltage stability analysis of power systems over a wide range of system conditions.

Dynamic analysis provides the most accurate indication of the time responses of the system. Therefore, Dynamic analysis is extremely useful for fast voltage collapse situations, such as loss of generation and system faults, especially concerning the complex sequence of events that lead to the instability. However Dynamic simulations fail to provide information such as

the sensitivity or degree of stability. More importantly, dynamic simulations are extremely time-consuming related to the CPU. Engineering resources are required for its computation and analysis, in the form of differential and algebraic expressions to quantify the phenomenon. [1]

Power system operation mainly depends on the interaction of three things power sources, loads and network. During a load pickup there are some events, which can induce voltage collapse via loss of a generating unit, a transmission line, or a transformer. Sometimes if the tap position setting of an OLTC is too low, it may create reverse instead of helping the system. In the case of generators, if the excitation hits its limit then it creates a considerable impact on the voltage stability.

The system stability mainly depends on its components performance for a sudden disturbance. The responsible components for the power system instability are non-linear e.g. generators, motors, load devices, tap changers (controllers), etc. System stability mainly depends on the interaction between the devices interconnected. That is why it is important to model all the components individually in order to have proper visualization about their performance. There are three ways to control voltage which are by adjusting the generator excitation, by using OLTC or by providing reactive power support. [1]

**2.2 Basic Principles of Active and Reactive Power Flow Control**

Active (real) and reactive power in a transmission line depend on the voltage magnitudes and phase angles at the sending and receiving ends as well as line impedance. To understand the basic concept behind the controllers a simple model is considered in Fig. 2.2.1. The sending and receiving end voltages are assumed to be fixed and can be interpreted as points in large power systems where voltages are "stiff". Assuming that the resistance of high voltage transmission lines are very small, there is equivalent reactance connected in between sending and receiving ends. The receiving end is modeled as an infinite bus with a fixed angle of 0°.

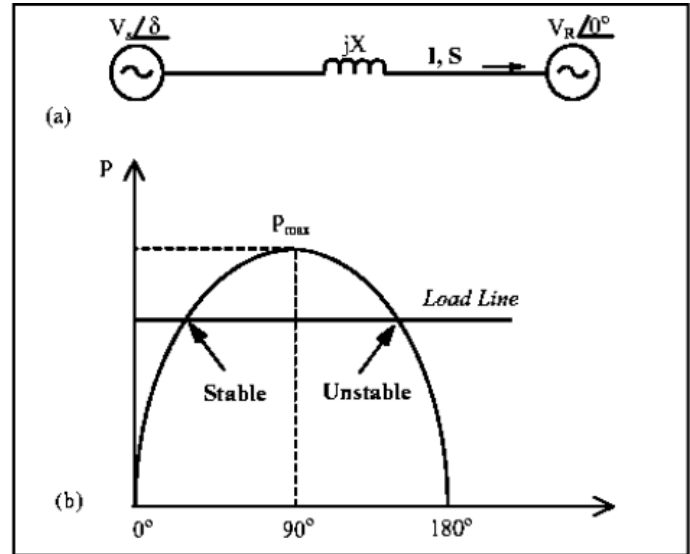


Figure 2.2.1: Model for Calculation of Real and Reactive Power flow

Similarly, for the sending end:

$$P_S = \frac{V_S V_R}{X} \sin \delta - P_{max} \sin \delta \quad \dots\dots\dots (2.2.4)$$

$$Q_S = \frac{V_S^2 - V_S V_R \cos \delta}{X} \quad \dots\dots\dots (2.2.5)$$

Where V<sub>S</sub> and V<sub>R</sub> are the magnitudes (in RMS values) of sending and receiving end voltages, respectively, where δ is the phase-shift between sending and receiving end voltages. [2, 3]

The system is assumed to be a lossless system and so the equations for sending and receiving active power flows, P<sub>S</sub> and P<sub>R</sub>, are equal. The maximum active power transfer occurs, for the given system, at a power or load angle δ equal to 90° which can be seen in the figure 2.2.1(b). Maximum power occurs at a different angle if the transmission losses are included. The system is stable or unstable depending on whether the derivative dP/dδ is positive or negative. The steady state limit is reached when the derivative is zero. [2, 3]

In practice, a transmission system is never allowed to operate close to its steady state limit, as certain margin must be left in power transfer in order for the system to be able to handle disturbances such as load changes, faults, and switching operations. The intersection between a load line representing sending end mechanical (turbine) power and the demand line defines the steady state value of δ. The angle can be increased by a small increase in mechanical power at the sending end. With increasing load demands the angle goes beyond 90° and results in less power transfer. This accelerates the generator and further increases the angle making the system unstable. However, the increased angle δ increases the electric power to correlate the mechanical increased power. The concepts of dy-

dynamic (small signal stability) or Transient (large signal stability) are used to determine the appropriate margin for the load angle  $\delta$ . [2, 3]

By the IEEE definition, "dynamic stability is the ability of the power system to maintain synchronism when subjected to a severe transient disturbance such as a fault or loss of generation". Typical power transfers correspond to power angles below  $30^\circ$ ; to ensure steady state rotor angle stability, the angles across the transmission system are usually kept below  $45^\circ$ . Inspecting the equations carefully reveals that the real or active power transfer depends mainly on the power angle and also reactive power requirements at sending and receiving ends. The both ends typically require high power transfers. Hence it is evident that the reactive power transfer depends mainly on voltage magnitudes, which flows from the highest voltage to the lowest voltage, while the direction of active power flow depends on the sign of the power angle. [2, 3]

Another interesting observation is the dependability on reactance. The maximum power transfer  $P_{max}$  and the angle between two ends vary on variation of reactance. The regulation of power flow is also possible by varying the sending and receiving end voltages. For a given power flow, a change of "X" also changes the angle between the two ends. Regulating the magnitudes of sending and receiving ends voltages,  $V_S$  and  $V_R$ , respectively, can also control power flow in a transmission line. From the equations of active & reactive power 2.2.4 & 2.2.5 respectively, it can be emphasised that the regulation of voltage magnitude has much more influence over the reactive power flow, than the active power flow. [2, 3]

### 2.3 Classification of Power System Stability

There are two types of power system stability rotor angle stability and voltage stability. The power system stability classified based on time scale and driving forces which is tabulated in table 2.3.1. Based on time scale, stability is divided into short-term (few seconds) and long term (few minutes) stability. Also stability is classified as load driven or generator driven based on the instability driving forces. [1]

The rotor angle stability is classified as small-signal and transient stability. The small signal stability deals with small disturbances in the form of un-damped electromechanical oscillations. The transient stability is initiated by large disturbances due to lack of synchronizing torque. The angle stability time frame is the electromechanical dynamics of the power system. The dynamics of the time frame typically last for a few seconds. For this reason, it is called short term time scale. Time scale of short-term voltage stability and rotor angle stability is the same. But sometimes it is difficult to differentiate between short-term voltage stability and rotor angle stability. There are two types of stability problems emerged in the long-term time scale based on frequency and voltage. The long-term voltage stability is characterized by the actions of the devices such as delayed corrective actions and load shedding. [6]

Table 2.3.1 Power System Stability Classification

Time scale	Generator-driven		Load-driven	
Short-term (few seconds)	Rotor angle stability		Short-term voltage stability	
	Transient	Small signal		
Long-term (few minutes)	Frequency stability		Long-term voltage stability	
			Small disturbance	Large disturbance

Table 2.3.2 Power System Component and Load Classifications

Time scale	System component	Type of load
Instantaneous	Network	Static loads
Short-term	Generators, Switching capacitors/reactors, FACTS,SVC.	Induction motors
Long-term	OLTC,OXL	Thermostatically controlled loads

Voltage stability is also termed as load stability because it occurs due to load dynamics. On the time scale of load dynamics voltage stability is divided into instantaneous, short term and long-term voltage stability. Table 2.3.2 shows the system components that affect the instantaneous, short-term and long-term stability. Network and static loads are known as instantaneous components of the system. Because they response immediately to changes occurred in the system. Short term voltage stability depends on the performance of the various components such as excitation of synchronous generator, induction motor, switching capacitors and static var compensators (SVC) and flexible AC transmission system (FACTS). Long-term voltage stability depends on the slow responding components such as OLTC, OXL, thermostatic loads. [1]

In order to analyze voltage stability, it is worth in segregating the voltage stability into small and large disturbances. Small disturbance voltage stability controls voltage after small disturbances, e.g. changes in load. The small disturbance voltage stability is investigated through steady state analysis. Here, the power system is linearized around an operating point. And the analysis is typically based on the eigenvalue and eigenvector techniques. Large disturbance voltage stability is associated with the response of the power system to large disturbances such as faults, or sudden loss of load or sudden loss of generation. Large disturbance voltage stability can be used to study non-linear time domain simulations in short-term time frame and

also in long-term time frame. Load flow analysis along with non-linear time domain simulations can be used to study large disturbance voltage stability. The combinations of both linear and non-linear tools are mainly used in a voltage stability problem. [1]

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## **2.4 Some of the Power System Voltage Collapse & Blackouts. [11]**

In 2012

On 14 January, a 380 kV transformer failure in Bursa Natural Gas Fueled Combined Cycle PP in Turkey, was accused of voltage deviations in the interconnected power grid that resulted in a blackout. Additionally, another failure occurred in 154 kV Babaeski substation caused blackout in Trakia. During the outage 6 cities in the Marmara Region of the country or more than 20 million people were affected. The power was back in all cities in the evening. The blackout knocked out metro and tram operation in Istanbul. Also gas heating systems didn't worked during the blackout. Industrial production was hurt badly too. The problem resolved by getting electricity from Bulgaria to Trakia and feeding lines in İstanbul from Ambarlı Natural Gas PP in İstanbul.

On 4 April, a blackout hit every city in Cyprus after the Dhekelia power station failed (with a lack of electric power from 04:42 to 09:20).

On 29 June, a line of thunderstorms with hurricane-force winds swept from Iowa to the Mid-Atlantic coast and knocked out power to more than 3.8 million people in Indiana, Ohio, West Virginia, Pennsylvania, Maryland, Virginia, Delaware, North Carolina, Kentucky, and metropolitan Washington, DC.

On 30 July, due to a massive breakdown in the northern grid, there was a major power failure which affected seven north Indian states, including Delhi, Punjab, Haryana, Himachal Pradesh, Uttar Pradesh, Jammu and Kashmir, and Rajasthan.

On 31 July, the July 2012 India blackout, which is being called the biggest ever power failure in the world, leaves half of India without electricity supply. This affected hundreds of trains, hundreds of thousands of households and other establishments as the grid that connects generating stations with customers collapsed for the second time in two days.

On 29-30 October, Hurricane Sandy brought high winds and coastal flooding to a large portion of the eastern United States, leaving an estimated 8 million customers without power. The storm, which came ashore near Atlantic City, New Jersey as a Category 1 hurricane, would ultimately leave scores of homes and businesses without power in New Jersey (2.7 million), New York (2.2 million), Pennsylvania (1.2 million), Connecticut (620,000), Massachusetts (400,000), Maryland (290,000), West Virginia (268,000), Ohio (250,000), and New Hampshire (210,000). Power outages were also reported in a number of other states, including Virginia, Maine, Rhode Island, Vermont, and the District of Columbia.

In 2013

On 8-9 February, some 650,000 homes and businesses in the northeastern US lost power as the result of a powerful nor'easter that brought hurricane-force wind gusts and more than two feet of snow to New England.

On the first of April 2013, 100 000 people in Poland suffered under power outages due to heavy snow falls. Warsaw Airport found the snow a bit difficult to operate in.

## **2.5 Various Voltage Stability Analysis Methods**

Voltage stability is a attempt of load dynamics to restore power consumption beyond the capability of the combined transmission and generation system. The controllers have their own limitations. Voltage can be maintainable at desired limits under normal situation. In a particular situation arising during operation of the power system viz. major outages or large demand, the controllers may reach their functional limits. The voltage stability problem associated in such case requires proper attention when there is increase in demand load and exploitation of the power transmission system. An abrupt voltage collapse can happen. Hence, continued monitoring of the system state is required. The main reason behind such collapse is insufficient reactive power at the weak buses. An additional reactive power support may be fed to the system either by fixed or switched capacitors, the voltage instability may be overcome. [1]

In a optimal impedance solution, a voltage collapse proximity indicator on power system load buses has been proposed. The indicator performance is investigated for two types of load increment, viz. the load increase at a particular bus and the load increase throughout the system. During a single load variation in the system the indicator can provide a good indication about the maximum possible power that could be delivered to the load. On the other hand, the indicator does not accurately predict the maximum possible power when the load in the entire system is increased than the single load variation. [4]

The method exist which identifies those areas that experience voltage collapse and also identifies the equipment outages that cause sectional voltage collapse. This method determines the series of events that cause the voltage collapse a contingency due to clogging voltage instability, Stemming from an increased transfer, wheeling or load pattern or loss of control voltage instability owed to equipment outages. This method requires little computation and is comprehensive in finding all regions/sections with voltage collapse in each region. In large AC/DC systems the implementation of both point of collapse (POC) and continuation is another method for the computation of voltage collapse points. [4]

There are other methods that analyze voltage stability, such as, model analysis using snapshots, test function, bifurcation theory, energy function methods, bus participation method, singular value method, optimization techniques, quasi steady-state method and the index method.

Tracking stability margins is a demanding problem because of its nonlinearity nature. The SMART Device estimates the proximity of voltage collapse. In this method we use local



measurements like bus voltage and load currents. This method determines the relative strength and weakness of the transmission system connected to a particular load bus precisely an evaluation method. It produces an estimation of the strength/weakness of the transmission system connected to the estimated transmission capacity based on local measurement. SMART Device is the stability monitoring and reference setting device. The operating principle is, at the voltage collapse point the magnitude of the Thevenin impedance is equal to the magnitude of the load apparent impedance. Thevenin impedance is the Thevenin equivalent of the network as seen from the local substation. In this method Thevenin equivalent impedance is obtained from some locally measured data. [1]

The system operates within an adequate security margin by estimating the maximum permissible loading of the system. In-order to determine the maximum permissible load (static voltage stability limit) of a power system P-V and Q-V curves are used. Bonneville power administration uses the conventional P-V and Q-V curves as a tool for assessing the voltage stability of the system. The P-V and Q-V curves are nonlinear at the maximum permissible power point. At maximum permissible power point the gradient of the curves changes sign. Therefore, without practically generating the entire curves the critical load estimation by using information at a particular operating point may not lead to the correct result. To overcome this, a simple method has been proposed to estimate the critical load at the verge of voltage collapse based on V-I characteristic. This method requires present data for bus voltage and current. It also requires some past operating points of the system. These bus voltage and current data are readily available in all power system to prepare the V-I characteristic. The voltage and current data are processed through the least squares method to generate the V-I characteristic. The extrapolated part of the characteristic is then used to estimate the critical load at the verge of voltage collapse. This method does not require the knowledge on other system parameters or system wide information, which makes easy to use this method. [5]

A stability factor method identifies the critical lines instead of critical buses of a power system. The stability factor method was compared with three prevailing established methods. The first method is the Lee's method of stability. It uses stability margin as voltage stability criterion to determine whether the system is stable. The bus that has a stability margin closer to zero is considered as critical bus. The second method is the Kessel's stability indices method. This method computes the stability index of each bus in the system and identifies the high index value bus as critical ones. The third method is the Schluster's stability indicators method which is developed based on the changes in the load flow Jacobian. The stability indicator is a measure of the proximity to voltage collapse. It is determined from the eigenvalues of the load flow Jacobian. The eigenvalues are estimated for all load buses. The buses should have larger eigenvalue in a secure voltage control area. For the critical bus the eigenvalue decreases to less than unity which could be the origin of voltage collapse. [1]

Global positioning systems (GPS) can economically synchronize the sampling process at a distant substation. Phasor measuring unit (PMUs) is the basic hardware box that converts current and voltage signals into complex phasors. It is a mature tool now which uses synchronization signals from the GPS satellite systems. Protection and control systems limit the impact, stop degradation and restore the system to a normal state at the time of major disturbance by using appropriate corrective actions when needed. Wide area measurement and protection system recognize, propose and execute the coordinated stabilizing actions, which helps to limit severity of disturbances. A system design has been proposed based on the synchronized phasor measurement units, encouraging system protection schemes for frequency, angle and voltage instabilities.

There is no simple way to identify the location of the critical node and the critical transmission path. Some of the methods check the system's Jacobian matrix to determine the critical node. This needs computation to estimate real-time voltage stability. However, voltage phasors contain enough information to detect the voltage stability margin of a power system. To identify the critical transmission paths with respect to the real or reactive power loading based on the voltage phasors approach, a voltage collapse proximity index has been proposed. In this method, the difference between the halved voltage phasor magnitudes of relevant generator considered as transmission path stability index as well as the voltage drop along the transmission path. Two types of transmission paths were proposed. Those are active transmission path and reactive power transmission path. Active transmission path is a sequence of connected buses with declining phase angles starting from a generator bus. Reactive power transmission path is a sequence of connected buses with declining voltage magnitudes again starting from a generator bus. In this method, the power transfer on that transmission path becomes unstable due to voltage collapse if the value of transmission path stability index reaches zero. [5]

## 2.6 Influence of Different Power system components on Dynamic Voltage Stability

Usually slower acting devices and fast-acting devices contribute to the evolution of voltage collapse due to sudden disturbances in the power system. On load taps changers, generator over excitation limiters, characteristics of the system loads are considered slower acting devices. Induction motors, excitation system of synchronous machines and compensation devices are considered fast acting devices. [7]

Tap changers in main power delivery transformers are the main mechanisms to operate and regulate the voltage automatically. Tap changers control the voltage by changing the transformer turns ratio. In many cases, the variable taps are placed on the high voltage side of the transformer. The reason for that it is easier to communicate. Various acronyms have been suggested for the transformer tap changer mechanisms. Those are on load tap changers (OLTC), under load tap changers (ULTC), tap changers under load (TCUL), and load tap changers (LTC). [7]

There are two types of tap changer models which are very common to use. Those are continuous type and discrete type. Continuous models are based on the assumption of continuously changing taps. Discrete models are based on the discontinuous or step-by-step tap change. A typical transformer equipped with an OLTC feeds the distribution network and maintains constant secondary voltage. OLTC operates with a certain delay. It depends on the difference between the reference and actual voltages at OLTC input. The phenomenon of raising the position of on-load tap changer for raising the secondary voltage causes the drop of secondary voltage. The secondary voltage of a transformer maintains a level higher than its lower bound by automatic OLTC even if the voltage of primary transmission system drops. However, the secondary voltage becomes unstable if the load demand becomes excessive. The instability of tap changer happen when the load demand is increased other than kept constant. The reverse action of the tap changer could occur when the initial operating voltage in the secondary side of the transformer is far less than the rated value. The effects of OLTC transformer on voltage stability and the identification of the critical OLTC transformer in a general power system have been studied. [7]

Synchronous generator is the primary device for voltage and reactive power control in power system. The most important reactive power reserves are located in the synchronous generator. Active and reactive power delivering capabilities of generator are required to achieve the best results in voltage stability studies. The generator may lose their ability to act as a constant voltage source because of the high reactive power demand by the loads and the field current limits. For such case the generator terminal voltage reduces and it behaves like a voltage source behind the synchronous reactance. In 1986, K. Walve first suggested the effect of excitation system limits on voltage stability. The power system may become unstable due to lack of reactive resources if the generator hits the reactive power limits. The reactive power output of a generator reaching a limit has two causes: excitation current limit and the stator current limit. Two types of excitation current limits are over excitation and under excitation limit. To avoid stator overloading, stator current limit is used to limit reactive power output. But the action of the stator current limit is not good for voltage stability. The stator current limiter decreases the reactive power capability to avoid stator overheating. As a result voltage decreases dramatically. It is important for voltage stability to have enough buses in a power system where voltage may keep constant. The excitation/automatic voltage regulator (AVR) system limits of synchronous generators are the most important for fulfilling that need. The AVR keep voltage constant. In some cases field current limitation introduces slow generator dynamics that interact with the long-term dynamic devices, such as OLTCs. In other cases the generator dynamics remain fast even after the limitation of rotor current. [3]

Loads are the driving force of voltage instability. So, voltage stability is also called load instability. Loads are aggregation of many different devices in the power system. For this

reason it is very difficult to model exact loads. The main problem is to identify the load composition at a given time. The differential equations for induction motors, tap changing near static load and heating system are non-linear. It is very difficult to parameterize for model estimation. A nonlinear model was proposed based on the assumption of exponential recovery. To obtain the dynamic voltage stability limit of a power system the first-order variable admittance model and the aggregate nonlinear recovery model have been considered along with the system dynamic equations. Third order induction motor model is another model to represent the induction motor loads. [1]

A short circuit in a network reduces the voltage. It also reduces the electrical torque developed by an induction motor. As a result the motor decelerate occurs. The speed reduction or slip increase of induction motor depends on the mechanical torque demand and motor inertia. During the short circuit, induction motors absorb a greater amount of reactive power. It operates at low factors which may further decrease the voltage and finally stall the motors.

From the viewpoint of dynamic phenomena, the voltage collapse starts locally at the weakest bus and spreads out to the other weak buses. [1]

### 3 MATHEMATICAL MODEL

Voltage instability is a structural instability of system caused by variation of numerous parameters. This chapter describes the mathematical model that is used in voltage stability studies and the assumptions that are used in this project's analysis.

#### 3.1 Determination of LVSI

##### 3.1.1 Two-Bus System

Fig 3.1.1 shows a simple two bus system where the source bus 'i' is connected to the load bus 'j' through a transmission line. It has an impedance of  $Z_{line}$ . The current I flows through the line as well as through the load impedance ( $Z_l$ ). The complex voltages considered as  $V_i \angle \delta_i$  and  $V_j \angle \delta_j$ , respectively at buses 'i' and 'j' [1]

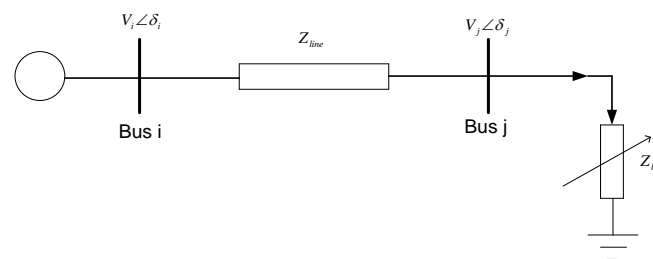


Figure 3.1.1 Simple two bus system to determine LVSI

According to the maximum power transfer theorem, when the magnitude of load impedance ( $Z_l$ ) becomes equal to the magnitude of the line impedance ( $Z_{line}$ ), the system reaches the maximum power point or the critical point at which the voltage collapse occurs. Thus, at voltage collapse point

$$Z_{line} = Z_l \quad \dots\dots\dots (3.1.1.1)$$

The magnitude of voltage drop across the transmission line is less than the magnitude of load bus voltage under normal condition. When the system reaches its maximum power transfer level, the magnitude of voltage across the transmission line becomes the same as the magnitude of load bus voltage. Therefore within the voltage stability limit, the relationship between the load voltage and voltage drop can be written as

$$|V_i - V_j| \leq |V_j| \quad \dots\dots\dots (3.1.1.2)$$

Equation (3.1.1.2) is in the form of complex variables. The magnitude form of equation 3.1.1.2 is

$$V_i^2 + V_j^2 - 2V_iV_j \cos(\delta_i - \delta_j) \leq V_j^2 \quad \dots\dots\dots (3.1.1.3)$$

By solving the inequalities:-

$$V_i^2 - 2V_iV_j \cos(\delta_i - \delta_j) \leq 0 \quad \dots\dots\dots (3.1.1.4)$$

Or

$$2 \frac{V_j}{V_i} \cos(\delta_i - \delta_j) - 1 \geq 0 \quad \dots\dots\dots (3.1.1.5)$$

At no load condition,  $V_i = V_j$  and angle  $\delta_i = \delta_j$ . Thus, the left hand side (LHS) of equation (3.1.1.5) becomes unity. Under normal operation (between no load and the maximum load) LHS of equation (3.1.1.5) will be greater than zero but less than unity. At the maximum loading condition (voltage collapse) it becomes zero. From the above reasoning, the voltage stability

index of the line at bus 'j' (LVSI<sup>j</sup>) can be expressed as follows

$$LVSI_j = 2 \frac{V_j}{V_i} \cos(\delta_i - \delta_j) - 1 \quad \dots\dots\dots (3.1.1.6)$$

Similarly, LVSI<sup>i</sup> at bus 'i' can be expressed as

$$LVSI_i = 2 \frac{V_i}{V_j} \cos(\delta_j - \delta_i) - 1 \quad \dots\dots\dots (3.1.1.7)$$

The magnitude of LVSI<sup>j</sup> and LVSI<sup>i</sup> depends on the direction and the amount of power flow.

### 3.1.2 Two-Bus System with a off-nominal Tap setting Transformer

Consider an off nominal tap setting transformer with an impedance of ( $Z_T$ ) is connected between bus 'i' (source bus) and bus 'j' (load bus). Fig. 3.1.2.2 shows the connection.  $Z_{ij}$ ,  $Z_{ij1}$ ,  $Z_{ij2}$  are equivalent mutual impedance, shunt impedance on side 'i' and side 'j' respectively in the equivalent  $\pi$  circuit model of the transformer which is shown in the Fig. 3.1.2.1 If the off-nominal turns ratio of the transformer is a: 1,

$Z_{ij}$ ,  $Z_{ij1}$  and  $Z_{ij2}$  are given

$$Z_{ij} = (a)Z_T \quad \dots\dots\dots (3.1.2.8)$$

$$Z_{ij1} = \left( \frac{a^2}{1-a} \right) Z_T \quad \dots\dots\dots (3.1.2.9)$$

$$Z_{ij2} = \left( \frac{a}{a-1} \right) Z_T \quad \dots\dots\dots (3.1.2.10)$$

When the off nominal turns ratio of the transformer

is 1: a,  $Z_{ij}$ ,  $Z_{ij1}$ ,  $Z_{ij2}$  become

$$Z_{ij} = (a)Z_T \quad \dots\dots\dots (3.1.2.11)$$

$$Z_{ij1} = \left( \frac{a}{a-1} \right) Z_T \quad \dots\dots\dots (3.1.2.12)$$

$$Z_{ij2} = \left( \frac{a^2}{1-a} \right) Z_T \quad \dots\dots\dots (3.1.2.13)$$

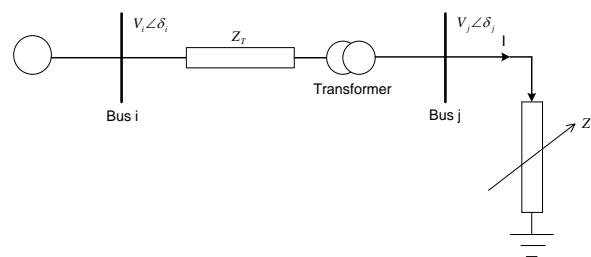


Figure 3.1.2.1 Simple two-bus system with transformer having off nominal turns ratios

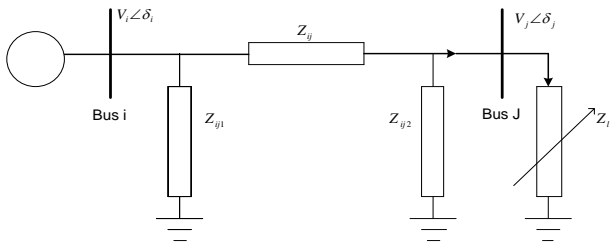


Figure 3.1.2.2 Equivalent circuit of Fig 3.1.2.1

Replace the generator and the transformer by Thevenin equivalent circuit. The source at bus 'i' is considered as ideal i.e. constant voltage with zero source impedance. With an off nominal transformer the parameters of the Thevenin equivalent circuit ( $Z_{th}$ ,  $V_{th}$  and  $\delta_{th}$ ) with turns ratio a: 1 are

$$Z_{th} = Z_{ij} \parallel Z_{ij2} = \frac{Z_{ij} Z_{ij2}}{Z_{ij} + Z_{ij2}} \quad \dots\dots(3.1.2.14)$$

$$V_{th} = \frac{V_i}{Z_{ij} + Z_{ij2}} Z_{ij2} = \frac{V_i}{a} \quad \dots\dots(3.1.2.15)$$

When the effect of transformer resistance is neglected, the angle  $\delta_{th}$  will be the same as  $\delta_i$

$$\delta_{th} = \delta_i \quad \dots\dots (3.1.2.16)$$

Similarly for transformers with 1: a off nominal turns ratio, the expression for  $Z_{th}$  and  $\delta_{th}$  remain the same but  $V_{th}$  is changed to

$$V_{th} = \frac{V_i}{Z_{ij} + Z_{ij2}} Z_{ij2} = aV_i \quad \dots\dots (3.1.2.17)$$

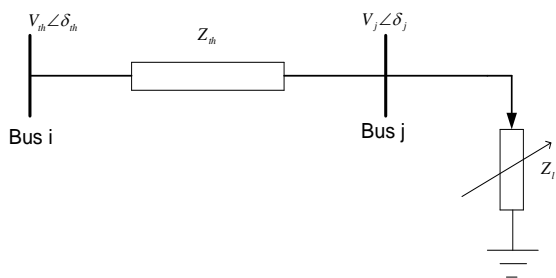


Figure 3.1.2.3: Equivalent circuit of Figure 3.1.2.2

The Thevenin equivalent circuit of the system (fig 3.1.2.2) is shown in fig 3.1.2.3. It is similar to fig 3.1.2.1. By replacing

$V_i$  by  $aV_i$  for 1:a off nominal turns ratio or  $\frac{V_i}{a}$  for a:1 off-nominal turns ratio  $LVSI_j$  and  $LVSI_i$  of fig 3.1.2.3 can be evaluated for equations (3.1.1.6) and (3.1.1.7) respectively. [1]

**3.1.3 LVSI of a transmission line in a General Power System**

Equations (3.1.1.6) and (3.1.1.7) don't require the generator, load and line parameters. Those equations only require the complex bus voltage to evaluate the line voltage stability index. This simple requirement can be used to evaluate the voltage stability index of a transmission line in a general power system as shown in Fig. 3.1.3.1 It requires only the complex voltage at buses 'i' and 'j' at both ends of the line. LVSI at bus 'j' side and LVSI at bus 'i' side can be determined using the expressions (3.1.1.6) and (3.1.1.7) for the transmission line between buses 'i' and 'j' which is shown in Fig 3.1.3.1.

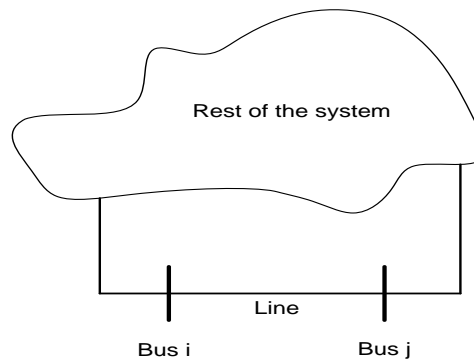


Figure 3.1.3.1 Transmission line connected between buses 'i' and 'j' in a general power system

**3.1.4 Determination of VSI of a General Power System**

Power system networks are mesh type thus it is important to determine the VSI of a mesh network. First compute the LVSI of the network at both ends of all branches (lines and transformers) using load flow results. Power flows from higher LVSI to lower LVSI in a branch. Higher LVSI side considered as stronger side or upstream side while the lower LVSI side considered as weaker side or downstream side. Based on the  $LVSI_j$  and  $LVSI_i$ , the mesh network decomposed into a number of power flow paths.[1]

Identification of power flow path starts at a source bus or upstream side and proceed to all downstream side buses which are connected through a branch. The upstream side has higher value than that at the downstream side. If the branch has lower LVSI at the upstream side than that at the downstream side, it should not be considered in the path. The above process is to be continued until it is found that no additional branch can be added to the path because of having lower LVSI at the upstream side compared to the downstream side. [1]



Start the identification of power flow paths at bus 1 which is connected to bus 2, bus 3 and bus 8 via lines  $L_1$ ,  $L_2$  and  $L_{11}$  respectively. Line  $L_1$  has a LVSI of 0.9881 (near bus 1) and 0.9504 (near bus 2). Since LVSI in the upstream side (bus 1) is higher than that at the downstream side (bus 2), the line should be included in the path. Similarly line  $L_2$  and  $L_{11}$  should be included in the path. Now start at bus 2 which is connected to bus 4 and bus 6 through  $L_{13}$  and  $L_{10}$ . These lines have higher LVSI at the upstream side compared to the downstream side thus these lines should be included in the path. Bus 6 connected with bus 10 through  $L_6$  which has a LVSI of 0.9988 (near bus 6) and 0.9967 (near bus 10). Since LVSI in the upstream side (bus 6) is higher than that at the downstream side (bus 10), the line should be included in the path. Now bus 4 is connected to bus 5 through  $L_4$  that has lower LVSI (0.9871) at the upstream side (bus 4) compared to the downstream side (1.0011 at bus 5) thus it should not be included in the path. In this case path terminates at bus 4 as shown as Fig. 4.1. This technique is to be repeated to identify the other possible power flow paths of the system. After identifying the all possible power flow paths need to calculate PVSI of each power flow path. PVSI can be written as

$$PVSI = \prod_{k=\zeta} LVSI_{kj} \dots\dots\dots (3.1.4.1)$$

Where  $\zeta$  is a set of lines that constitute a power flow path and j is the downstream side of the line. PVSI considered as the most heavily loaded path or critical path that is vulnerable to voltage collapse. The value of PVSI of the most heavily loaded path is considered as the overall stability index of the system. [1]

The voltage stability index (VSI) of the power system is expressed as follows

$$VSI = \min(PVSI_m) \dots\dots\dots (3.1.4.2)$$

Where m varies from 1 to n and n is total number of possible power flow paths originating from buses.

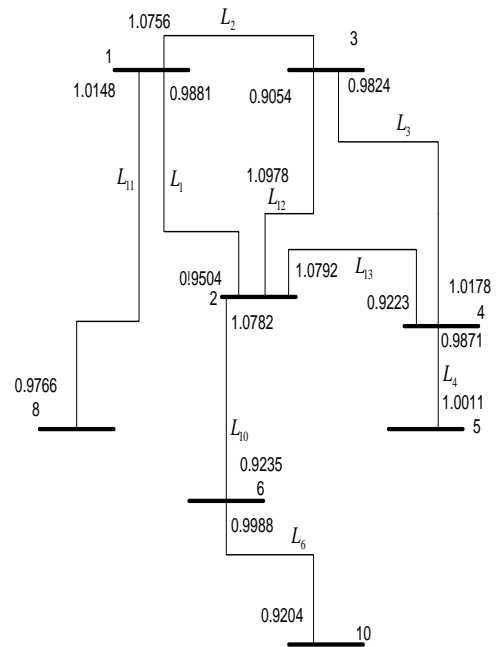


Figure 3.1.4.1: Power flow path identification

#### 4 APPLICATION OF MATHEMATICAL MODEL

Voltage magnitudes and angles of all the buses of the test system at base condition are given in table 4.1. Using the result of base case load flow the LVSI at both ends of all branches are computed through equations (3.1.1.6) and (3.1.1.7) and the values found are mentioned in Fig. 4.1.

Table 4.1 IEEE 10 Bus System, Bus Voltage and Angle at Base Load

Number	PU Volt	Angle (Deg)
1	1.05	12.24
2	1.04	2.17
3	0.99118	0.66
4	1	0.59
5	1.00353	-3.82
6	1.00054	0.7
7	1.04	-3.31
8	1.04	16.02
9	1	14.22
10	1	-2

Path no	Bus Number in the Power Flow Path						
P1	1	2	3	8	9	10	
P2	1	2	3	8	6	4	10
P3	1	2	3	8	4		

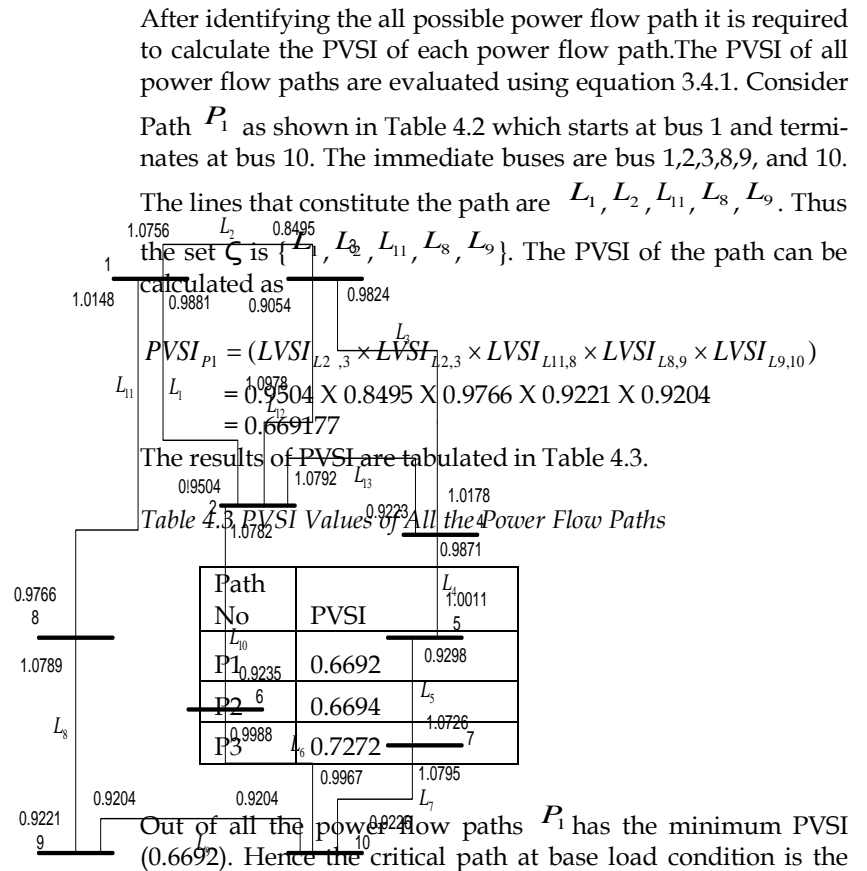


Figure 4.1 IEEE-10 bus test system

The above technique which discussed at section 3.1.4 is used repeatedly to identify the other possible power flow paths of the system. All power flow paths that start at bus 1 are given in Table 4.2.

Table 4.2 Power Flow Paths Starting From bus 1 at Based Load

Table 4.4 Values of All the Lines in the Identified Critical Power Flow Path

Line K	LVS <sub>Iki</sub>	LVS <sub>Ikj</sub>	LL <sub>k</sub>
L1	0.9881	0.9504	0.0377
L2	1.0756	0.8495	<b>0.2261</b>
L11	1.0148	0.9766	0.0382
L8	1.0148	0.9221	0.1568
L9	0.9204	0.9204	0
L13	1.0792	0.9223	0.1569

## 5 CONCLUSION

The voltage stability of the system can be expressed in voltage stability index of the system. The voltage stability index (LVSI) is calculated for all lines. The LVSI is compounded to determine path voltage stability index (PVSI) through possible power flow path of all permutation in the network. The power flow with minimum (PVSI) is the critical power flow path of the system and the critical line is demarcated by LVSI of all lines in the critical power flow path.

## APPENDIX

### [A] Glossary of Terms

*System* – A combination of generation, transmission, and distribution elements.

*Reliability* – A measure of how often electrical service is interrupted.

*Load* – An amount of end-use demand.

*Grid* – Usually used to describe the interconnected transmission system, although sometimes used with distribution (distribution grid) to describe the distribution system.

*Electricity* – The flow of electrons through a conductor.

*Generation* – The creation of electricity.

*Current* – The rate of flow of electrons through a conductor.

*Demand* – The total amount of electricity used at any given moment in time, usually measured in KW or MW

*Deregulation* – The process of decreasing or eliminating government regulatory control over industries and allowing competitive forces to drive the market.

*Distribution* – The delivery of electricity over medium and low-voltage lines to consumers of the electricity.

*Base load* – Electricity usage that is constant through a specified time period. Also used to refer to the generating units that run all 24 hours of the day to serve a system's base load demand.

*Blackout* – The loss of power to a portion of the distribution or transmission system.

*Circuit* – A complete path through which electricity travels, comprised of sources of electrons, energy consuming devices and conductors.

*Circuit breaker* – A device that interrupts electricity flow to a circuit by isolation the circuit from the source of electricity.

*Stability Limit* – The maximum power flow possible through some particular point in the system while maintaining stability in the entire system or the part of the system to which the stability limit refers.

*Stability* – The ability of an electric system to maintain a state of equilibrium during normal and abnormal conditions to which the stability limit refers.

*Disturbance* – 1. An unplanned event that produces an abnormal system condition. 2. Any perturbation to the electric system.

### [B]

Base case data Load

Number	Load MW	Load Mvar
1		
2	40	20

3	110	40
4	100	30
5	150	40
6	150	60
7	180	0
8		
9		
10	150	0

### Generator

Number	Gen MW	Gen Mvar
1	186.11	9.67
2	50	133.78
3		
4	80	38.36
5		
6		
7	192.14	86.34
8	198.93	49.28
9	100	-27.88
10	100	-3.11

### Transmission Lines

From Number	To Number	R	X	B
1	2	0.02	0.12	0.06
1	3	0.08	0.24	0.05
8	1	0.03	0.25	0
2	3	0.06	0.18	0.04
2	4	0.06	0.18	0.04
2	5	0.04	0.12	0.03
2	6	0.02	0.06	0.05
3	4	0.01	0.03	0.02
4	5	0.08	0.24	0.05
7	5	0.02	0.06	0.04
6	7	0.08	0.24	0.05
8	6	0.02	0.2	0
6	10	0	0.2	0
10	7	0.08	0.24	0.05
8	9	0.03	0.15	0
9	10	0.05	0.35	0

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